Engineering Notes

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Calculation of Vortex Sheet Roll-up in a Rectangular Wind Tunnel

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THE possibility of calculating the roll-up of a vortex sheet wake in a rectangular wind tunnel with solid walls was earlier demonstrated by Hackett and Evans,[1,2] who represented the wind tunnel effect by a few layers of image vortex sheets. The present Note utilizes the more general concept of the influence (Green's) function, analogous to that used in Ref. 3. For simplicity, the method is applied to a vortex sheet in the time dependent y, z plane, proven by Betz[4] to preserve all essential features of the roll-up process, at least in the far field. The computations are performed by the discretization of the continuous vortex sheet model by finite length elements which show more stable behavior than Westwater's[5] array of point vortices. It appears from the present calculations that for a suitable distribution of vortex sheet elements and for appropriate time increments, the roll-up process can be followed over sufficiently large times without the need to introduce artificial viscosity[6,7] or smoothing.[8]

Assuming the wind-tunnel walls to be solid and the flow symmetrical about the z-axis, Fig. 1, we solve for the complex disturbance velocity w in the rectangle $0<y<B$, $|z|<H/2$, satisfying the boundary conditions

$$Re\ w=0 \qquad y=0,B \qquad Im\ w=0 \qquad z=\pm H/2 \tag{1}$$

By the principle of linear superposition w can be represented by the contour integral

$$w(r)=\int_c \gamma(\chi)g(r,\rho)\,d\chi \tag{2}$$

where

$$r=y+iz \tag{3}$$

$$\rho=\eta+i\zeta \tag{4}$$

C is the half of the vortex sheet and

$$d\chi=(d\eta^2+d\zeta^2)^{1/2} \tag{5}$$

its line element.

The real-valued function γ is the vortex density function (streamwise vorticity shed by the half wing). According to Prandtl's high-aspect ratio wing theory, γ relates to the spanwise distribution of bound circulation Γ through

$$\gamma(y)=d\Gamma(y)/dy \tag{6}$$

Load distributions satisfying $\Gamma(y)=\Gamma(-y)$ and $\Gamma(b)=0$ where b is the span of the half wing, can be represented by the

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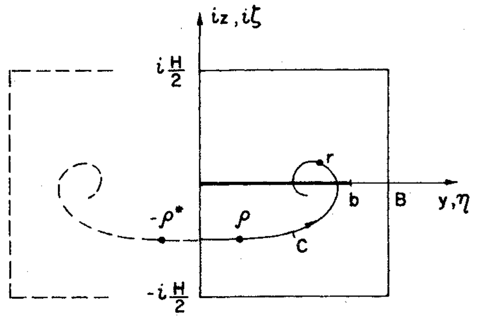

Fig. 1 Coordinate system.

series

$$\Gamma=b\sum_{n\in I} a_n \sin(n\theta) \tag{7}$$

where I is a set of odd natural numbers and

$$\theta=\arccos(y/b) \tag{8}$$

The influence function g can conveniently be written in the form

$$g(r,\rho)=f(r,\rho)+h(r,\rho) \tag{9}$$

The first term is the free air fundamental solution

$$f(r,\rho)=\frac{i}{2\pi}\left[\frac{1}{r-\rho}-\frac{1}{r+\rho^*}\right] \tag{10}$$

which describes the complex disturbance velocity at the observation point r due to a pair of point vortices of strengths $+1$ and -1, located at ρ and $-\rho^*$. (The asterisk indicates complex conjugation.) The second term h is an analytic function in the rectangle $0<y<B$, $|z|<H/2$, and represents the boundary effect of wind-tunnel walls.

For $r\in C$, indicating a point on the vortex sheet (tangential discontinuity), the integral of Eq. (2) is interpreted as the Cauchy principal value. Using the method of images,[3] the function g satisfying Eqs. (1, 9, and 10) is found to be

$$g(r,\rho)=\frac{i}{2H}\sum_{m=-\infty}^{\infty}\begin{aligned}&\{\exp[\pi(r-\rho-2mB)/H]-1\}^{-1}\\ +&\{\exp[\pi(r-\rho^*-2mB)/H]+1\}^{-1}\\ -&\{\exp[\pi(r+\rho^*-2mB)/H]-1\}^{-1}\\ -&\{\exp[\pi(r+\rho-2mB)/H]+1\}^{-1}\end{aligned} \tag{11}$$

The terms in braces represent the sums of singularities in the z-direction[3] so that we are left with only a single summation in the y-direction (horizontal reflections).

The numerical solutions are obtained by approximating the contour C by N straight-line elements. Denoting by r_j the midpoint of the jth element, and by ρ_j and ρ_{j+1} its end points we obtain

$$d\chi=d\rho(\rho^*_{j+1}-\rho^*_j)/|\rho_{j+1}-\rho_j|$$

$$=d\rho^*(\rho_{j+1}-\rho_j)/|\rho_{j+1}-\rho_j|$$

Assuming that over the j th element the functions γ and h have constant values γ_j and $h(r_k,r_j)$, respectively, and per-

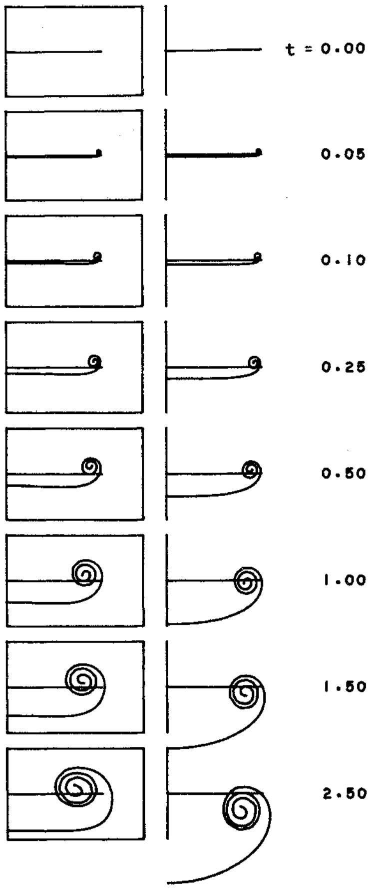

Fig. 2 Time history of a vortex sheet roll-up, $\Gamma/b = \sin\theta$.

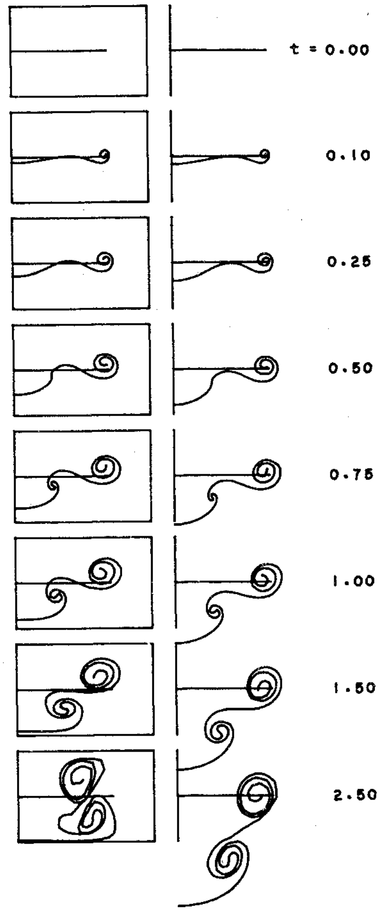

Fig. 3 Time history of a vortex sheet roll-up, $\Gamma/b = \sin\theta + [(2)^{1/2}/10]\sin 5\theta$.

forming the integration in Eq. (2) we obtain

$$w(r_k) = -\frac{i}{2\pi}\left\{\gamma_k \frac{\rho_{k+1}-\rho_k}{|\rho_{k+1}-\rho_k|}\log\frac{r_k+\rho^*_{k+1}}{r_k+\rho^*_k} + \sum_{j=1,j\neq k}^{N}\gamma_j\left[\frac{\rho_{j+1}-\rho_j}{|\rho_{j+1}-\rho_j|}\log\frac{r_k+\rho^*_{j+1}}{r_k+\rho^*_j} + \frac{\rho^*_{j+1}-\rho^*_j}{|\rho_{j+1}-\rho_j|}\log\frac{r_k-\rho_{j+1}}{r_k-\rho_j}\right]\right\} + \sum_{j=1}^{N}\gamma_j|\rho_{j+1}-\rho_j|\,h(r_k,r_j) \tag{12}$$

Denoting by superscripts s the coordinates in the sth time step, the following scheme is adopted‡

$$\rho^{s+1}_{k+1} = \rho^s_{k+1} + \tfrac{1}{2}[w(r^s_{k+1}) + w(r^s_k)]^*\Delta t^s \tag{13}$$

$$r^{s+1}_k = \tfrac{1}{2}(\rho^{s+1}_{k+1} + \rho^{s+1}_k) \tag{14}$$

where Δt^s is the time increment in the $s+1$th time step.

By the law of conservation of vorticity

$$\gamma^{s+1}_k = \gamma^s_k\,|\rho^s_{k+1}-\rho^s_k|\,/\,|\rho^{s+1}_{k+1}-\rho^{s+1}_k| \tag{15}$$

To save in computation time, the time increment is adjusted according to

$$\Delta t^{s+1} = \Delta t^s \max_k|\gamma^s_k|/\max_k|\gamma^{s+1}_k| \tag{16}$$

Examples calculated by the presently described method are exhibited in Figs. 2 and 3. Vortex roll-up histories in free air are compared with those in a solid wall wind tunnel for the following case: $B/b = 1.43$, $H/b = 0.95$. This corresponds to related experiments being currently conducted at the Carleton University low-speed wind tunnel.

Figure 2 shows results for an elliptic load distribution, $\Gamma/b = \sin\theta$ using 50 elements distributed according to $\rho^0_k/b = \sin[(\pi/2)\ (k-1)/50]$, and choosing $\Delta t^0 = 0.0005$. To reach the "time" $t = 2.5$, 100 time steps were required in each case. With the exception of the downwash, the roll-up patterns seem to differ very little in the initial stage. Later, however, the wind-tunnel spiral tends to deform (flatten in this case) according to the constraining walls.

The wind-tunnel distortion effect is more pronounced in Fig. 3, obtained for the load distribution[9] $\Gamma/b = \sin\theta + [(2)^{1/2}/10]\sin 5\theta$ (10% induced drag penalty). The function $|\gamma|$ has a local minimum at $y/b \cong 0.78$, and as predicted by Donaldson et al.[10] and Yates,[11] a double roll-up vortex is developed. The computations are performed using 75 elements, $\rho^0_k/b = \sin[(\pi/2)\ (1.01^{k-1}-1)/(1.01^{75}-1)]$, and choosing again $\Delta t^0 = 0.0005$. In this case 80 time steps were needed to reach $t = 2.5$.

Analysis of the data obtained confirms a significant feature of the vortex sheet roll-up in a closed-wall wind tunnel, namely that the vorticity distribution has essentially a stationary center of gravity. This is in marked contrast with the free air case where the center of gravity, while preserving its spanwise coordinate, moves downwards (negative z-direction).

‡In Sept. 1975 the authors learned from P. T. Fink of the University of New South Wales, Australia, of the "stepwise vorticity rediscretization method" which is similar to the present approach.[12]

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Effect of Compression Ratio on NO_X Production by Gas Turbines

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IT is well established as a result of correlations by Lipfert,[1] supported by Bahr[2] and by Nelson,[3] that the mass fraction of NO_x in the exhaust gases of currently operational aircraft gas turbines increases dramatically with compressor outlet temperature, hence with compression ratio. Sawyer et al.[4] and Ferri and Agnone[5] prefer a correlation in terms of maximum combustion temperature, but for the rather narrow range of equivalence ratios in the primary zones of existing combustors these correlations appear to be essentially equivalent.

The purpose of this Note is to point out that these correlations do not necessarily imply that a higher NO_x fraction in the exhaust must be accepted as a result of increased compression ratio and turbine inlet temperature, even for currently available burner technology.

As Lipfert[1] and Sawyer et al.[4] point out, the data correlations imply near constancy of the residence times in the primary zones of the burners for the engines correlated. The dramatic rise in NO_x with increasing compressor outlet temperature then follows from an increase in the rate of NO_x formation roughly according to the relation

$$(NO_x) \propto p_3^{1/2} e^{-2400/T_3} \tau_p$$

where p_3 and T_3 are compressor outlet pressure and temperature and τ_p is the residence time in the primary zone.

Note, however, that since the rates of the combustion reactions also are increased as T_3 and p_3 increase, it should be possible to decrease τ_p as T_3 and p_3 are increased. Indeed, correlations of combustor efficiency, η_b, for fixed geometry combustors have indicated a correlation of the form[6]

$$\eta_b(p_3^{1.75} e^{T_3/b}/m)$$

where m is mass flow through the burner, and b varies from 300 K for a fuel air ratio of 0.016 to 150K for 0.010. Taking $\tau_p \propto p_3/m$, i.e., neglecting small temperature changes in the primary zone, and assuming $p_3 \propto T_3^{(\gamma-1)/\gamma}$ we find that for *constant combustion efficiency*

$$(NO_x) \propto e^{-[(2400/T_3)+(T_3/b)]}/T_3^{0.25\gamma/(\gamma-1)} \quad (\eta_b = \text{constant})$$

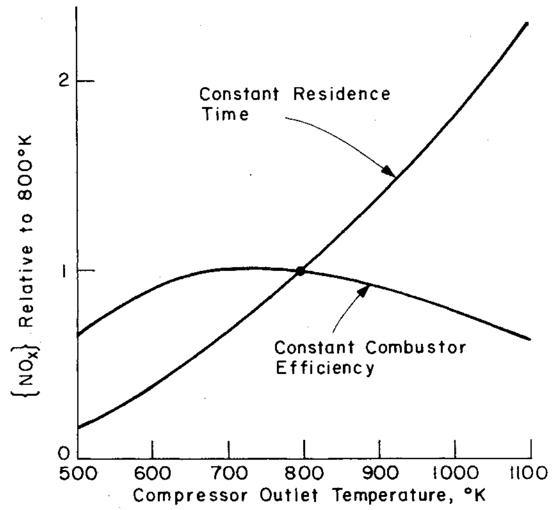

Fig. 1 Comparison of the variations of NO_x mass fraction with compressor outlet temperature which should result for burners having constant combustion efficiency, with that for burners having constant primary zone residence time.

This result is plotted in Fig. 1, compared to the corresponding result for τ_p = constant. We see that the NO_x mass fraction is sensibly constant over the range of T_3 from 600 to 900K, and is even predicted to fall off at higher compressor outlet temperatures. Hence the conclusion that for *constant combustion efficiency in a given combustor design,* NO_X *is nearly independent of combustor inlet temperature.*

The author is aware that the requirement for low CO and hydrocarbon efflux at part throttle conditions dictates high combustion efficiency at these conditions, and that combustion efficiency tends to decrease with decreased throttle setting for fixed geometry combustors. The part throttle requirements can be and have been met with fixed geometry combustors by choosing residence times longer than are required for full power operation. This solution is attractive in part because combustor length and volume are not as critical in large high pressure ratio engines as they were in early small turbojets.

It is suggested therefore, that NO_x production in high-pressure ratio engines can be reduced by designing for the minimum primary zone residence time consistent with combustion efficiency at full power, and then achieving satisfactory part throttle combustion efficiency by some scheme such as compressor outlet bleed or combustor bypass.

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